AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Pure Core 4

Friday 17 June 2016 Afternoon Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Express $\dfrac{19x-3}{(1+2x)(3-4x)}$ in the form $\dfrac{A}{1+2x}+\dfrac{B}{3-4x}$.

[3 marks]

(b) (i) Find the binomial expansion of $\dfrac{19x-3}{(1+2x)(3-4x)}$ up to and including the term in x^2.

[7 marks]

(ii) State the range of values of x for which this expansion is valid.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 1

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 1**

Do not write outside the box

2 By forming and solving a suitable quadratic equation, find the solutions of the equation

$$3\cos 2\theta - 5\cos\theta + 2 = 0$$

in the interval $0° < \theta < 360°$, giving your answers to the nearest $0.1°$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 2

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 2**

Do not write outside the box

3 (a) Express $\dfrac{3 + 13x - 6x^2}{2x - 3}$ in the form $Ax + B + \dfrac{C}{2x - 3}$.

[4 marks]

(b) Show that $\displaystyle\int_3^6 \frac{3 + 13x - 6x^2}{2x - 3}\,\mathrm{d}x = p + q\ln 3$, where p and q are rational numbers.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 3**

Do not write outside the box

4 The mass of radioactive atoms in a substance can be modelled by the equation

$$m = m_0 k^t$$

where m_0 grams is the initial mass, m grams is the mass after t days and k is a constant. The value of k differs from one substance to another.

(a) (i) A sample of radioactive iodine reduced in mass from 24 grams to 12 grams in 8 days.

Show that the value of the constant k for this substance is 0.917004, correct to six decimal places.

[1 mark]

(ii) A similar sample of radioactive iodine reduced in mass to 1 gram after 60 days.

Calculate the initial mass of this sample, giving your answer to the nearest gram.

[2 marks]

(b) The half-life of a radioactive substance is the time it takes for a mass of m_0 to reduce to a mass of $\frac{1}{2}m_0$.

A sample of radioactive vanadium reduced in mass from exactly 10 grams to 8.106 grams in 100 days.

Find the half-life of radioactive vanadium, giving your answer to the nearest day.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 4**

Do not write outside the box

5 It is given that $\sin A = \frac{\sqrt{5}}{3}$ and $\sin B = \frac{1}{\sqrt{5}}$, where the angles A and B are both acute.

(a) (i) Show that the exact value of $\cos B = \frac{2}{\sqrt{5}}$.

[1 mark]

(ii) Hence show that the exact value of $\sin 2B$ is $\frac{4}{5}$.

[2 marks]

(b) (i) Show that the exact value of $\sin(A - B)$ can be written as $p(5 - \sqrt{5})$, where p is a rational number.

[4 marks]

(ii) Find the exact value of $\cos(A - B)$ in the form $r + s\sqrt{5}$, where r and s are rational numbers.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

6 The line l_1 passes through the point $A(0, 6, 9)$ and the point $B(4, -6, -11)$.

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} -1 \\ 5 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$.

(a) The acute angle between the lines l_1 and l_2 is θ.

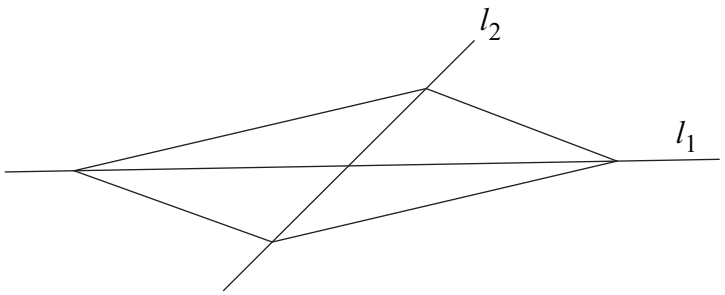
Find the value of $\cos\theta$ as a fraction in its lowest terms.

[5 marks]

(b) Show that the lines l_1 and l_2 intersect and find the coordinates of the point of intersection.

[5 marks]

(c) The points C and D lie on line l_2 such that $ACBD$ is a parallelogram.



The length of AB is three times the length of CD.

Find the coordinates of the points C and D.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 6**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 6**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 6**

Do not write outside the box

7 A curve C is defined by the parametric equations

$$x = \frac{4 - e^{2-6t}}{4}, \qquad y = \frac{e^{3t}}{3t}, \qquad t \neq 0$$

(a) Find the exact value of $\frac{dy}{dx}$ at the point on C where $t = \frac{2}{3}$.

[5 marks]

(b) Show that $x = \frac{4 - e^{2-6t}}{4}$ can be rearranged into the form $e^{3t} = \frac{e}{2\sqrt{(1-x)}}$.

[2 marks]

(c) Hence find the Cartesian equation of C, giving your answer in the form

$$y = \frac{e}{f(x)[1 - \ln(f(x))]}$$

[2 marks]

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

8 It is given that $\theta = \tan^{-1}\left(\frac{3x}{2}\right)$.

(a) By writing $\theta = \tan^{-1}\left(\frac{3x}{2}\right)$ as $2\tan\theta = 3x$, use implicit differentiation to show that $\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{k}{4+9x^2}$, where k is an integer.

[3 marks]

(b) Hence solve the differential equation

$$9y(4+9x^2)\frac{\mathrm{d}y}{\mathrm{d}x} = \operatorname{cosec} 3y$$

given that $x = 0$ when $y = \frac{\pi}{3}$. Give your answer in the form $\mathrm{g}(y) = \mathrm{h}(x)$.

[7 marks]

QUESTION PART REFERENCE	**Answer space for question 8**

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 8**

END OF QUESTIONS

Copyright information

For confidentiality purposes, from the November 2015 examination series, acknowledgements of third party copyright material will be published in a separate booklet rather than including them on the examination paper or support materials. This booklet is published after each examination series and is available for free download from www.aqa.org.uk after the live examination series.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team, AQA, Stag Hill House, Guildford, GU2 7XJ.

Copyright © 2016 AQA and its licensors. All rights reserved.